

Phase Noise in RF and Microwave Amplifiers

Rodolphe Boudot and Enrico Rubiola

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FEMTO-ST Institute
CNRS and Université de Franche Comté, Besançon, France

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Abstract

Understanding the amplifier phase noise is a critical issue in numerous fields of engineering and physics, like oscillators, frequency synthesis, telecommunications, radars, spectroscopy, in the emerging domain of microwave photonics, and in more exotic domains like radio astronomy, particle accelerators, etc.

This article analyzes the two main types of phase noise in amplifiers, white and flicker. Consequently, the phase-noise power spectral density is $S_\varphi(f) = b_0 + b_{-1}/f$. Phase noise results from adding white noise to the RF spectrum around the carrier. For a given amount of RF noise added, b_0 is proportional to the inverse of the carrier power P_0 . By contrast, b_{-1} is a constant parameter of the amplifier, in a wide range of carrier power. Accordingly, the flicker phase noise b_{-1}/f is independent of P_0 . This fact has amazing consequences on different amplifier topologies. Connecting m equal amplifiers in parallel, b_{-1} is $1/m$ times that of one device. Cascading m equal amplifiers, b_{-1} is m times that of one amplifier. Recirculating the signal in an amplifier so that the gain increases by a power of m (a factor of m in dB) due to positive feedback (regeneration), which for integer m is similar to the case of m amplifiers, we find that b_{-1} is m^2 times that of the amplifier alone.

Starting from the fact that near-dc flicker exists in all electronic devices, although generally not accessible from outside, the simplest model for the $1/f$ phase noise is that the near-dc $1/f$ noise phase-modulates the carrier through some parametric effect in the semiconductor. This model predicts the behavior of the (simple) amplifier and of the different amplifier topologies. Numerous measurements on amplifiers from different technologies and frequencies (HF to microwaves), also including some obsolete amplifiers, validate the theory.

1 Introduction

Low phase noise amplification is crucial in a variety of applications. In the oscillator, the phase noise of the sustaining amplifier is converted into frequency noise through the Leeson effect [1, 2, 3, 4], for the oscillator phase fluctuation, which is the integral of frequency, diverges in the long run. In turn, the oscillator noise impacts on the bit error rate [5, 6] and on security [7] in communications, and on radars [8, 9]. Doppler radars require ultra-low phase noise to avoid that the oscillator noise sidebands exceed the echo signal. Low phase noise amplification is important per se in precise synchronization systems, independently of the oscillator, because phase represents time. Finally, the book [10] provides a useful panorama, though not up to date.

The $1/f$ phase noise in amplifiers is still a controversial subject. There is no doubt that in the absence of a carrier the noise at the output of an amplifier is nearly white. Near-dc $1/f$ noise, discovered in the early times of electronics [11], is now considered an ubiquitous phenomenon that escapes from unified understanding. In the case of electronic components, most models resort to two original articles [12, 13]. Therefore, phase flicker is a cyclostationary process originated by near-dc to $1/f$ noise brought to the vicinity of the carrier [14]. Non-linear noise modeling is a difficult challenge because the model rely on the identification of the near-dc noise sources, which can in turn be non-linear [15] or associated to a non-linear circuit element [16]. Since the conversion of near-dc noise into phase noise is generally not implemented in commercial CAD programs, the simulation may require dedicated software. Despite these models are not a perfect representation of the device physics, some of them provide results in quite a reasonable agreement with the measured phase noise [16, 17, 18]. Some theoretical models, supported by experiments, provide useful information about amplifier $1/f$ phase noise for several technologies [19, 20, 21, 22]. The semiconductor-physics approach [23] and the microscopic models that follow, despite their potential accuracy, are too complex and difficult to use.

Surprisingly, little information is available about the consequences of the more or less understood phase noise mechanisms on the amplifier and on more complex amplifier architectures. This article is intended to fill this gap, providing insight, practical knowledge, design rules and experimental confirmation.

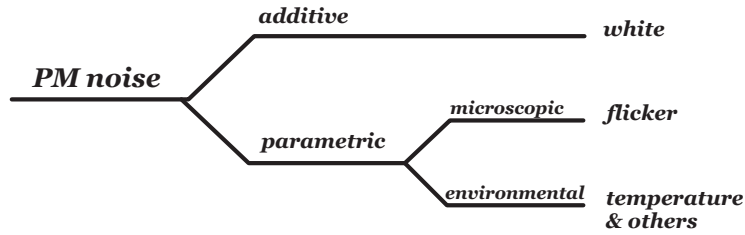


Figure 1: Amplifier internal phase noise mechanisms.

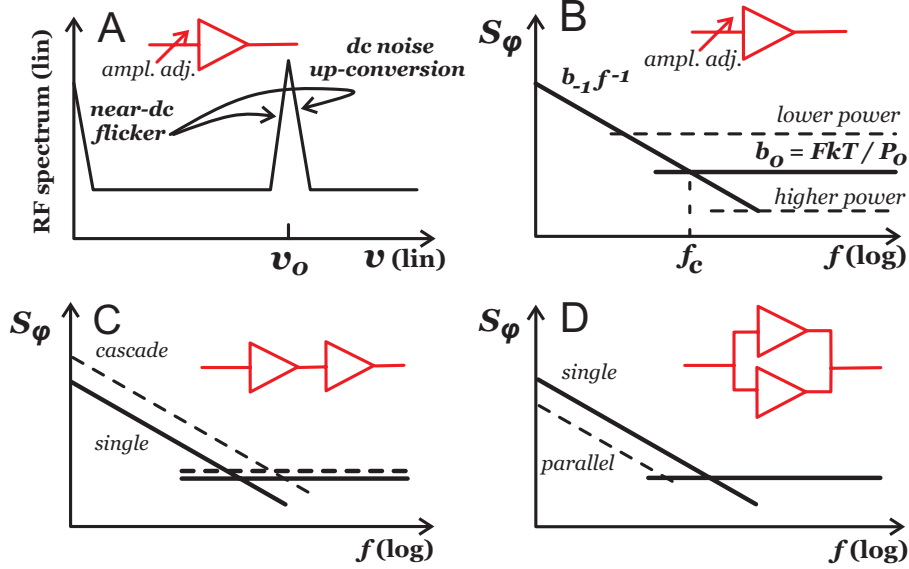


Figure 2: Phase noise rules for several amplifier topologies. A: Noise up-conversion from near-dc the carrier frequency, which originates $1/f$ phase noise. B: Single Amplifier. C: Cascaded Amplifiers. D: Parallel Amplifiers.

2 Phase Noise Mechanisms

Physical insight suggests that phase noise in amplifiers originates from two basic mechanisms, additive and parametric, as shown on Fig. 1. Restricting our attention to white and flicker noise, the phase noise spectrum $S_\varphi(f)$ is completely described by the first two terms of the power-law

$$S_\varphi(f) = b_0 + \frac{b_{-1}}{f} . \quad (1)$$

The white phase noise b_0 derives by adding to the carrier a random noise of power spectral density $N = FkT_0$, where k is the Boltzmann constant, and F is the amplifier noise figure defined at the reference temperature $T_0 = 290$ K (17 °C). It is useful to have on hand the following numerical values

$$kT_0 = 4 \times 10^{-21} \text{ J} \quad (-174 \text{ dBm/Hz}).$$

In modern low-noise amplifier F is typically of 1–2 dB. It may depend on bandwidth through the loss of the input impedance-matching network, and on technology. If the actual temperature is not close enough to T_0 the quantity F is meaningless, and the noise is described by $N = kT_e$, where T_e is the equivalent noise temperature, which includes amplifier and its input termination. We assume that N is independent of frequency in a wide range around the carrier frequency ν_0 , as it happens in most practical cases.

Adding N to a carrier of power P_0 results in random phase modulation of power spectral density

$$b_0 = \frac{FkT_0}{P_0} . \quad (2)$$

The above holds in the linear region of the amplifier. If the amplifier is operated in large-signal regime, where it is nonlinear or saturated, F may increase [24, 25].

At low frequencies, the amplifier phase noise is of the $1/f$ type, which currently referred to as flicker. Near-dc flicker noise takes place at the microscopic scale [12, 13], for little or no correlation is expected between different region of the device. This is supported by the fact that the probability density function is normal, which originates from the central-limit theorem in the presence of a large population of independent phenomena. Understanding phase flickering starts from the simple fact that the noise is white in the absence of a carrier. The heuristic proof given by Nyquist [26] for thermal noise is convincing also if the noise figure F is introduced, which is not necessarily a thermal phenomenon. As a matter of fact, close-in noise shows up only when the carrier is sent at the input. This means that phase flickering can only originate from up-conversion of the near-dc $1/f$ noise, as shown in Fig. 2 A. The noise up-conversion can be described as follows. We denote with $u(t) = U_0 e^{j2\pi\nu_0 t} + n'(t) + jn''(t)$ the input signal, where $U_0 e^{j2\pi\nu_0 t}$ is the ‘true’ (accessible) input and $n = n' + jn''$ the near-dc equivalent noise at the amplifier input; and with $v(t) = a_1 u(t) + a_2 u^2(t) + \text{noise}$ the output signal. The near-dc noise $n(t)$ is *not* the random signal that would ideally be measured with an oscilloscope. Instead, it is an abstract quantity with spectrum proportional to $1/f$ that accounts for the parametric nature of flicker. The amplifier is described as a (smooth) nonlinear function truncated at the second order, where the coefficient a_1 is the (usual) voltage gain denoted with A elsewhere in this article. Expanding $v(t)$ and selecting only the $2\pi\nu_0$ terms we get

$$v(t) = a_1 U_0 e^{j2\pi\nu_0 t} + 2a_2 [n' + jn''] U_0 e^{j2\pi\nu_0 t} , \quad (3)$$

from which

$$\alpha(t) = 2 \frac{a_2}{a_1} n'(t) \quad S_\alpha(f) = 4 \frac{a_2^2}{a_1^2} S_{n'}(f) \quad (4)$$

$$\varphi(t) = 2 \frac{a_2}{a_1} n''(t) \quad S_\varphi(f) = 4 \frac{a_2^2}{a_1^2} S_{n''}(f) . \quad (5)$$

Equations (3), (4) and (5) express the simple fact that the noise sidebands are proportional to the carrier amplitude, and therefore AM and PM noise are independent of the carrier amplitude or power. In this representation we use the nonlinearity, present in virtually all devices, to transpose the random signal $n(t)$. Of course a fully-parametric model yields the same results, at a cost of higher complexity.

Experiments show that below the compression point, b_{-1} is almost independent of the carrier power [18, 21, 27, 28]. The quasi-static perturbation technique provides fairly good agreement between simulated and experimental $1/f$ phase noise data in Si and SiGe amplifiers [17]. Other investigations describe the $1/f$ phase noise as a modulation from the near-dc $1/f$ current fluctuation in microwave HBT amplifiers [22] and in InGaP/GaAs HBTs [29]. The analysis of the literature cited indicates that, regardless of the theoretical approach and of the amplifier technology, the *amplifier behavior* is that of a linear phase modulator driven by a near-dc process

$$b_{-1} = C \quad (\text{constant, independent of } P_0) . \quad (6)$$

Neither the near-dc noise nor the modulation efficiency are affected by the carrier power unless the amplifier is pushed in the compression region. If this happens, the dc bias changes and in turn small changes of b_{-1} are expected in an unpredictable way. Our experiments, detailed in Sec. 4 confirm this behavioral model.

3 Analysis and design rules

3.1 Single Amplifier

The phase noise of an amplifier is shown in Fig. 2 B. Increasing the input power by a factor n the white noise decreases by a factor n , and the flicker corner frequency f_c increases by the same factor n . This fact can be used to reverse engineer oscillators from noise, identifying some relevant parameters like the resonator Q and driving power [2, Chap.6], [30].

It is worth pointing out that the flicker corner frequency f_c sometimes found in the amplifier specifications is misleading because f_c follows the input power according to

$$f_c = \frac{b_{-1}}{FkT_0} P_0 , \quad (7)$$

instead of being an independent parameter. In SPICE and in some other CAD programs the flicker is described in terms of f_c , which for the same reason is an unfortunate choice.

3.2 Cascaded Amplifiers

When several amplifiers are cascaded (Fig. 2 C), the equivalent noise figure of the chain is given by the Friis formula [31]

$$F = F_1 + \frac{F_2 - 1}{A_1^2} + \frac{F_3 - 1}{A_1^2 A_2^2} + \frac{F_4 - 1}{A_1^2 A_2^2 A_3^2} + \dots , \quad (8)$$

where A_i is the voltage gain of the i -th stage, so A_i^2 is the power gain. The Friis formula expresses the fact that the noise of the first stage is $F_1 kT_0$, including the input termination, and that the noise $(F_i - 1)kT_0$ of the i -th stage ($i \geq 2$) is divided by the power gain of the $i - 1$ preceding stages. By virtue of (2), the obvious extension of the Friis formula to phase noise is

$$b_0 = \left[F_1 + \frac{F_2 - 1}{A_1^2} + \frac{F_3 - 1}{A_1^2 A_2^2} + \frac{F_4 - 1}{A_1^2 A_2^2 A_3^2} + \dots \right] \frac{kT_0}{P_0} . \quad (9)$$

In most practical cases the noise of the chain is chiefly determined by the noise of the first stage. This of course applies to the RF spectrum and for the phase noise spectrum.

Conversely the flicker phase noise is ruled by (6). Since the amplifier $1/f$ phase noises are statistically independent and independent of the carrier power, the $1/f$ noise of a chain of m amplifiers is

$$b_{-1} = \sum_{i=1}^m (b_{-1})_i \quad (10)$$

Accordingly when two or three equal amplifiers are cascaded, the phase noise flicker is 3 dB or 4.8 dB higher than that of the single amplifier.

Combining white (9) noise and flicker noise (10), we find the spectrum shown in Fig. 2 C.

3.3 Parallel Amplifiers (PA)

It was clear since the early time of electronics that two or more amplifiers (vacuum lamps) can be connected in parallel to increase the available power. From our standpoint we can define a *parallel amplifier* as an amplifier network in which m amplifier cells of the same gain share equally the burden of delivering the desired output power. Advantage can be taken from the appropriate choice of the power splitter and combiner used to couple the cells. The push-pull configuration uses 180° junctions. This type of symmetry suppresses the even-order harmonic distortion, which is most appreciated in audio applications. The balanced amplifier [32, Sec. 11.4] uses the symmetry of the 90° junction to improve input and output impedance matching. The distributed amplifier [32, Sec. 11.4], preferred when a wide frequency range is to be achieved at any cost, uses a series of taps in a delay line to put the cells at work.

For the sake of analysis simplification, we assume that

- the cells are equal, and have voltage gain A , input and output impedance R_0 and noise figure F ,
- the input power-splitter and the output power-combiner are loss-free¹ and impedance matched to R_0 .

Accordingly, the PA gain is equal to the gain A of a cell, and the compression power is m times the compression power of one cell. With our hypotheses the PA looks like the amplifier of Fig. 2 D, where only two cells are shown.

Denoting with P_0 the input power, the power at the input of each cell is P_0/m . Consequently the white phase noise is

$$(b_0)_{\text{cell}} = \frac{FkT_0}{P_0/m}$$

at the output of each cell, and

$$b_0 = \frac{FkT_0}{P_0} \quad (\text{PA}) \quad (11)$$

at the output of the parallel amplifier, after adding m signals of equal power, same statistical properties, and independent. This also means that the noise figure of the PA is equal to the noise figure F of one cell.

Recalling that $S_{n''}$, and $S_{n'}$ as well, has $1/f$ spectrum, the flicker noise can be derived from (5) applied to one cell

$$(b_{-1})_{\text{cell}} = 4 \frac{a_2^2}{a_1^2} S_{n''}(1 \text{ Hz}) .$$

¹In the case of the distributed amplifiers, it is conceptually impossible that all cells handle the same power. Yet this hypothesis helps to understand.

At a closer sight, one should introduce coupling coefficient κ_i and κ_o of the input and output couplers, and also the dissipative losses. Since the effect of the coefficient κ is an intrinsic power loss $1 - \kappa^2$ if the coupler, for the RA gain is reduced by a factor $\sqrt{(1 - \kappa_i^2)(1 - \kappa_o^2)}$. The small effect of the coupler losses will be neglected in the rest of this Section.

It is wise to adjust the phase for the roundtrip gain $A_0\beta$ to be real, hence G is real. Of course, it is to be made sure that $0 < A_0\beta < 1$. The condition $A_0\beta > 0$ means that the feedback is positive, while $A_0\beta < 1$ is necessary for the loop not to oscillate.

The equivalent noise temperature is the amplifier equivalent noise temperature referred back to the RA input, i.e., the temperature of the amplifier increased by the loss of the input coupler. The detailed analytical proof given in [35] for the Q -multiplier, which is an application of the RA where a resonator is inserted in the feedback, holds for the RA in the general case. The consequence is that the RA white noise is

$$b_0 = \frac{FkT_0}{P_0} + \text{losses} . \quad (15)$$

The flicker noise is best understood by replacing A_0 with $A_0 e^{j\psi}$, where $\psi(t)$ is the instantaneous value of the internal-amplifier noise. In practical design the flicker of phase shows up at low frequencies, at least a factor of 10^2 lower than the inverse of the roundtrip time. In these conditions the signal circulating in the loop sees a quasi-static phase ψ , hence the gain can be written as

$$A \rightarrow \frac{A_0 e^{j\psi}}{1 - A_0\beta e^{j\psi}} \quad (16)$$

and expanded using $e^x = 1 + x$ for low noise

$$A = \frac{A_0}{1 - A_0\beta} \left[1 + j \frac{1}{1 - A_0\beta} \psi \right] . \quad (17)$$

Accordingly the RA phase noise is

$$\varphi(t) = \frac{1}{1 - A_0\beta} \psi(t) \quad (18)$$

$$(b_{-1})_{\text{RA}} = \left[\frac{1}{1 - A_0\beta} \right]^2 (b_{-1})_{\text{ampli}} , \quad (19)$$

which after (14) is equivalent to

$$(b_{-1})_{\text{RA}} = m^2 (b_{-1})_{\text{ampli}} . \quad (20)$$

It is instructive to compare the $1/f$ of a cascade of m amplifiers to that of a RA. The comparison makes sense only if the two configurations use the same type of amplifier and have the same gain. The latter condition sets the value of β . It follows from (10) that the flicker of the cascade is

$$(b_{-1})_{\text{chain}} = m(b_{-1})_{\text{ampli}} , \quad (21)$$

thus

$$(b_{-1})_{\text{RA}} = m(b_{-1})_{\text{chain}} . \quad (22)$$

However amazing at first sight, this conclusion is not a surprise because in the chain the carrier is phase-shifted by m independent random processes, while in the RA it is shifted by the same process m times.

3.5 The virtues of the error amplifier

A side effect of Eq. (6) is that the amplifier noise sidebands are proportional to the carrier. Since the amplifier $1/f$ noise sidebands are proportional to the carrier, an error amplifier that receives the null signal of a bridge is virtually free from close-in flicker.

This will be exploited in the NMS² of Fig. 4(B), where the noise sidebands of the error amplifier (G) are referred to the DUT power. Therefore, the contribution of the error amplifier to the background b_{-1} is divided by the carrier suppression ratio, that is, approximately by the ratio between the DUT power to the residual carrier at the input of the error amplifier (the exact value accounts for the loss of the hybrid junction). This ratio can be of 60–100 dB.

If the DUT of Fig. 4(B) is an amplifier shared by the NMS and by an external circuit, we can use the NMS to null the amplifier noise in closed loop. In practice, the $1/f$ noise is limited by the background of the NMS. This is used for the reduction of the oscillator $1/f$ frequency noise [36, 37], with detectors conceptually equivalent to that of Fig. 4(B).

The idea of the error amplifier can be used in the feedforward amplifier to reduce the AM-PM noise [38], and for similar reasons also the harmonic distortion [39]. The latter is by the way the main raison d'être of the feedforward amplifier.

3.6 The effect of physical size

Physical insight suggests that the flicker coefficient b_{-1} is proportional to the inverse of the volume of the amplifier active region. This can be seen through a gedankenexperiment in which we set up a m -cell parallel amplifier, whose flicker is $b_{-1} = \frac{1}{m}(b_{-1})_{\text{cell}}$ [Eq. (12)]. Then we join the m cells forming a single large device trusting the fact that flicker is of microscopic origin and that the elementary volumes are uncorrelated. This assumption is supported by the fact that the variety of flicker models for specific cases share the common fact that flicker is of microscopic origin. Moreover, the sum of a large number of independent processes by virtue of the central-limit theorem yields a Gaussian distribution, which is generally observed.

Our inverse-volume law must be taken with prudence. First, for a given volume flicker depends on technology. Second, the volume law certainly breaks down at nanoscale, where the size is smaller than the coherence length of the flicker phenomenon and the elementary volumes are no longer independent; and likely also at large scale. Nonetheless, the inverse-volume law is a useful design guideline.

²We apologize for the forward reference to Sec. 4.1. Yet, this Section goes necessarily under ‘Analysis and design rules.’

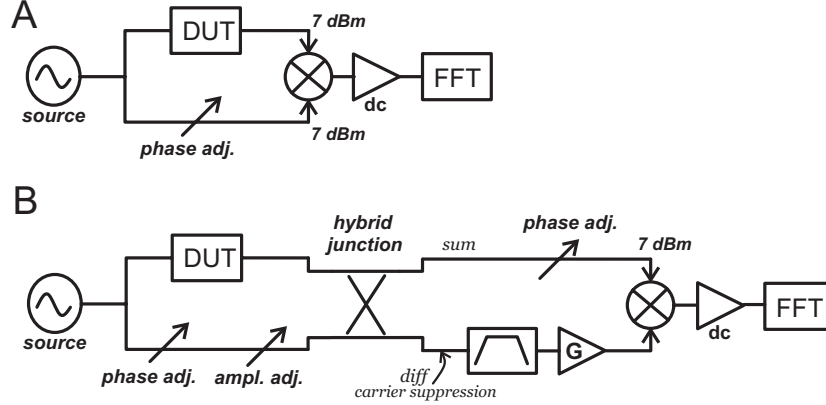


Figure 4: Phase noise measurement methods. A: saturated mixer. B: low-flicker carrier-suppression scheme.

4 Experimental Proof

4.1 Measurement method

Two different schemes, shown in Fig. 4, have been used to measure the amplifier phase noise, depending on needs. The scheme A is that of commercial phase noise measurement systems. A Schottky-diode double-balanced mixer saturated at both inputs with 7–10 dBm driving power is used as the phase detector. The two inputs are to be in quadrature. In this condition the mixer converts the phase difference φ into a voltage $V = k_d \varphi$ with a typical conversion factor of 100–500 mV/rad. The mixer output is low-pass filtered, amplified and sent to the FFT analyzer. The background $1/f$ noise is chiefly due the mixer. Typical values are of -140 dBrad²/Hz for RF mixers and -120 dBrad²/Hz for microwave mixers. The white part of the background noise is generally due to the dc-amplifier (1.5 nV/ $\sqrt{\text{Hz}}$) referred to the mixer input. Values of -155 to -170 dBrad²/Hz are common in average or good experimental conditions. The amplifier described in [40] is designed for this type of applications, and optimized for the lowest background flicker when connected to a 50Ω source.

SiGe amplifiers may exhibit outstanding low $1/f$ noise, below the instrument background. When this happens, the scheme of Fig. 4 A is replaced with that of Fig. 4 B. This detector, well known in the literature [41, 42, 43, 44], works as a Wheatstone bridge followed by a microwave amplifier and a synchronous detector. Since all the DUT noise is contained in the sidebands, low $1/f$ background is achieved by suppressing the carrier at the input of the microwave amplifier labeled G. The latter amplifies only the DUT noise sidebands, which are low-power signal, so that virtually no flicker up-conversion takes place. Microwave amplification before detecting has the additional advantage of low white background and reduction of 50–60 Hz spurs. This happens because the dc amplifier take in low-frequency magnetic fields, while microwave amplifiers do not. Neglecting dissipative losses, the white background is

$$(b_0)_{\text{bg}} = \frac{2FkT_0}{P_{\text{hyb}}} \quad (23)$$

Table 1: RF and microwave amplifiers tested in our laboratory.

Amplifier	Frequency (GHz)	Gain (dB)	$P_{1\text{dB}}$ (dBm)	F (dB)	DC bias	b_{-1} (meas.) (dBrad ² /Hz)
AML812PNB1901	8 – 12	22	17	7	15 V, 425 mA	–122
AML412L2001	4 – 12	20	10	2.5	15 V, 100 mA	–112.5
AML612L2201	6 – 12	22	10	2	15 V, 100 mA	–115.5
AML812PNB2401	8 – 12	24	26	7	15 V, 1.1 A	–119
AFS6	8 – 12	44	16	1.2	15 V, 171 mA	–105
JS2	8 – 12	17.5	13.5	1.3	15 V, 92 mA	–106
SiGe LPNT32	3.5	13	11	1	2 V, 10 mA	–130
Avantek UC573	0.01 – 0.5	14.5	13	3.5	15 V, 100 mA	–141.5

where F is the noise figure of the amplifier labeled G, P_{hyb} is the microwave power at the inputs of the hybrid junction, and the factor 2 is the junction intrinsic loss. The value of -185 dBrad²/Hz is easily achieved at 15 dBm power level. The $1/f$ background is not limited by necessary and known factors. We obtained $(b_{-1})_{\text{bg}} = -150$ dBrad²/Hz in the very first experiments [43], and $(b_{-1})_{\text{bg}} = -180$ dBrad²/Hz with a series of tricks [44]. The phase-to-voltage gain can be 40 dB higher than that of the saturated mixer. Interestingly, the scheme of Fig. 4 B can be built around a commercial instrument (Fig. 4 A), re-using mixer, dc amplifier, FFT and data acquisition system. The only problem with Fig. 4 B is that the carrier suppression must be adjusted manually, which may take patience, experimental skills, and often replacing some parts when frequency is changed.

4.2 Experimental results

We measured the amplifiers listed in Table 1. All are commercial products but the LPNT32, which was designed and implemented at LAAS, Toulouse [17]. We believe that the AML812PNB1901 and the AML812PNB2401, claimed to be ultra-low noise units by AML, are actually parallel amplifiers. The reason is that b_{-1} and compression power scale down by almost 3 dB per factor-of-two increase in the dc bias [2, Chapter 2]. Our measurements aim at the knowledge of the coefficient b_{-1} , and at the experimental confirmation of the behavioral rules stated in Section 3. The results are given as a series of spectra discussed underneath. Additionally, b_{-1} is reported on the right-hand column of Table 1.

White phase noise, though understood in the literature, is a necessary complement to this work and a sanity check for the results.

4.2.1 Phase noise of an amplifier

The first experiment is the simple measurement of the phase noise of several microwave amplifiers at different values of input power (Figure 5). It is clearly seen on all spectra that b_{-1} is independent of power, and that this fact is independent of technology and also holds in moderate compression operation. This confirms the parametric nature of flicker and validates the main point of the behavioral model. In the spectra of Fig. 5(a), 5(b) and 5(e), in the latter only for -45 and -40 dBm input power, the white noise b_0 follows exactly the $1/P_0$ law stated by Eq. (2). The white phase noise cannot be observed in Figures 5(c), 5(d) and 5(e) (-30 dBm input power) because the frequency span of our FFT analyzer is insufficient. The AFS6 measured at -25 dBm [green spectrum in Fig. 5(e)] is the only one curve of Fig. 5 that does not follow exactly the rules we stated. The bump between 100 Hz and 10 kHz seems to be due to saturation in an intermediate stage.

The AML812PNB1901 and the LPNT32 (Fig. 5(a) and 5(b), respectively) are intended for low phase noise applications and for high spectral purity oscillators [17, 46, 45]. These amplifiers exhibit $b_{-1} < 120$ dBrad²/Hz. The white noise of these amplifiers, though at first sight remarkably low, is the noise predicted by Eq. (2).

It is worth mentioning that the power efficiency (output power divided by dc-bias power) is of 50% for the LPNT32 (LAAS laboratory design [17]), and of 0.5%–2.5% for the other amplifiers (commercial units). Though one may object that the LPNT32 is a low-gain single-stage special-purpose amplifier, efficiency is surprisingly high. The $1/f$ noise of this amplifier indicates that low flicker design is not incompatible with efficiency.

Measuring one amplifier at different frequencies we observed no significant difference in noise (Fig. 5(f)). Since this result is consistent with our experience, we did not repeat systematically the test.

In the case of HF-VHF bipolar amplifier, the values of b_{-1} are clustered around -140 dBrad²/Hz. Fig. 6(a) (lowest plot) shows an example.

4.2.2 Cascaded amplifiers

In a second experiment we check on the rule of cascaded amplifiers versus Eq. 10 by connecting 2–3 equal units (Fig. 6). We did not insert attenuators in the

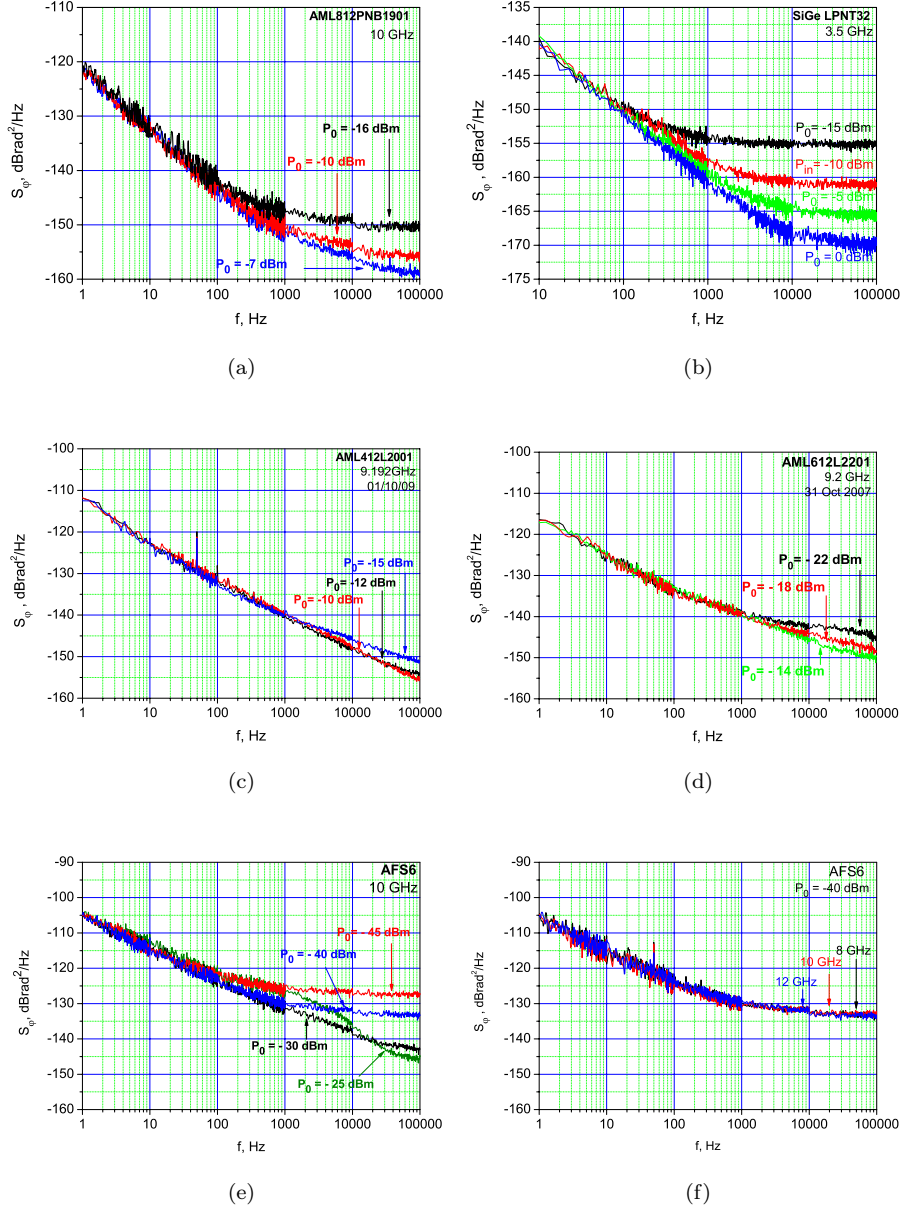


Figure 5: Phase noise of some amplifiers, measured at different input power and frequency. The plot 5(b) was measured at LAAS (Toulouse, France) using the system described in [24] (see also [45]).

chain. The consequence is that the input power must be scaled down proportionally to the total gain for the output to be kept in the linear or moderate-compression region. Yet, impedance matching is improved with microwave isolators.

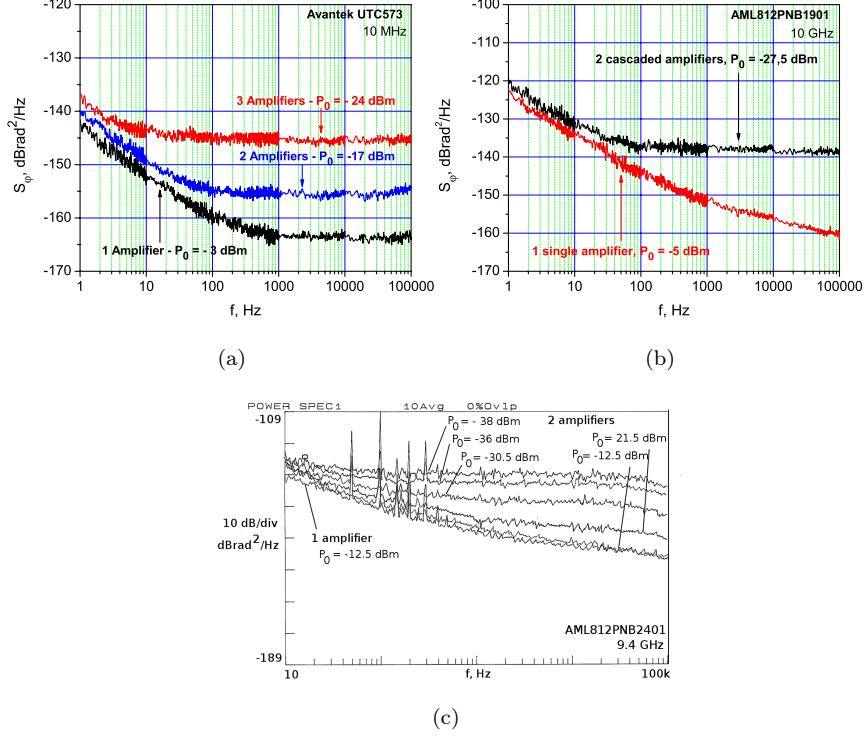


Figure 6: Phase noise of cascaded amplifiers, compared to the noise of a single amplifier.

Figure 6(a) shows the phase noise of a chain consisting of 1–3 UTC573 operated at 10 MHz. The measured flicker fits almost exactly the model, which predicts an increase of 3 dB for 2 cascaded units, and of 4.8 dB for 3 units. The small discrepancy is ascribed to the difference between the amplifiers. The reference (one amplifier) is the noise of a single device instead of the average of the 2–3 amplifiers. For the single amplifier measured at -3 dBm input power, the white noise hits the background of the instrument. Otherwise it follows Eq. (9). The same experiment made on one or two AML812PNB1901 at 10 GHz gives the same result (Fig. 6(b)).

Figure 6(c) shows the phase noise of a pair of cascaded AML812PNB2401, measured at low input power and compared to the single amplifier. The flicker coefficient is $b_{-1} = -119 \text{ dBrad}^2/\text{Hz}$ for one amplifier, and $-116.5 \text{ dBrad}^2/\text{Hz}$ for the two amplifiers, independent of power. The reason for thorough noise investigation in the microwatt range of input power is that this amplifier is an important piece of the frequency-synthesis chain used at SYRTE for fundamental metrology [47], and at some point the amplifier was suspected to flicker more than expected when used at low power. In this chain, built around a femtosecond laser that delivers a 250 MHz frequency comb at the output of a photodetector, the amplifier receives the power of some -30 dBm associated to a spectral line.

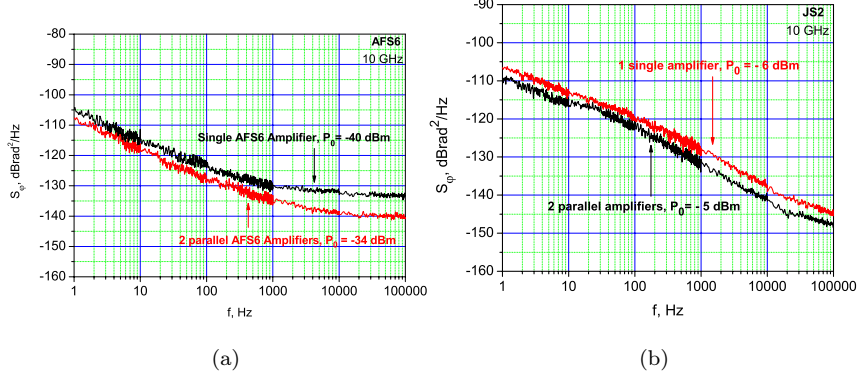


Figure 7: Phase noise of parallel amplifiers, compared to the noise of a single amplifier.

4.2.3 Parallel amplifiers

In a third experiment we measured the phase noise of a pair of amplifiers (AFS6 or JS2) connected in parallel in order to reduce the flicker noise (Fig. 7). We used Wilkinson power splitters/combiners at the input and at the output instead of 90° couplers for the trivial reason that layout and trimming were simpler with the parts we had on hand. Anyway, the demonstration of our ideas is independent of the impedance-matching benefit of the 90° couplers. The power P_0 refers to the main input, before splitting the signal. Measuring the AFS6, we had to adapt the power to experimental needs, while the JS2 could be measured at about the same level for the single amplifier and for the parallel configuration.

In both cases we observe that the flicker of the pair is 2.5 dB lower than the noise of the single amplifier, while the model predicts 3 dB. This is ascribed to the gain asymmetry and to the asymmetry of the power splitter and combiner.

In Figure 7(b) we observe a significant discrepancy with respect to the power-law (1). A slope of -7 dB/decade shows up in the left-hand side of the spectrum, up to 10–30 Hz, followed by a small bump. We are aware that this behavior is quite unusual, and we have no explanation. However, the amplifiers are in a good shape and the spectrum is reproducible.

4.2.4 Regenerative amplifier

The fourth experiment is best described by first explaining how the RA came to this work. In quite a different research program, Kirill Volyanskiy was investigating on high-spectral-purity photonic oscillators, in which the microwave frequency is set by the delay of an optical fiber [49, 50, 48]. Out of a temporary lack of parts he used regeneration to “double” (in dB) the gain of an AML812PNB1901 as a replacement for two cascaded amplifiers, and eventually he used the roundtrip time as the bandpass filter that selects the oscillation frequency among the multiples of the base frequency set by the fiber length (50 kHz for 4 km length with refraction index of 1.5, whose delay is $\tau = 20 \mu\text{s}$). Kirill came with the spectrum of Fig. 8 asking for interpretation. Unfortunately,

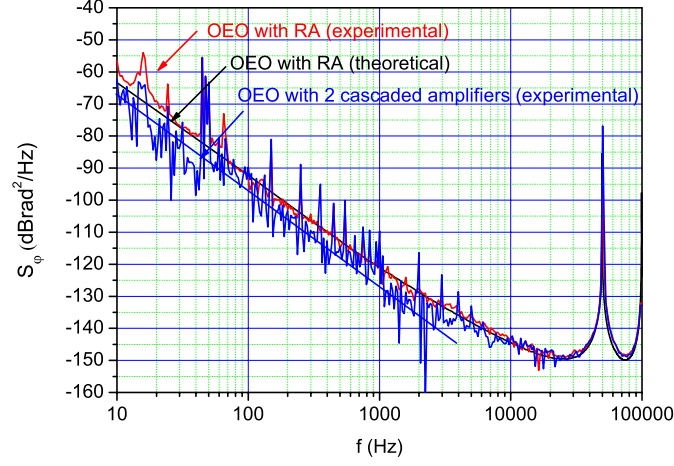


Figure 8: Phase noise of an opto-electronic oscillator (OEO). Two types of sustaining amplifiers are considered: (1) two AML812PNB1901 cascaded, and (2) a single AML812PNB1901 whose gain is “doubled” (in dB, so that it is equivalent to two amplifiers cascaded) using positive feedback. Courtesy of Kirill Volyanskiy [48].

this happened when the other measurements were finished, and for practical reasons we could not measure the RA alone as we did with the other amplifier configurations. However, the result fits so well in the theory that it deserves to be included as it is.

The relevant fact is that cascading two equal amplifiers yields a $1/f$ noise 3 dB higher than the noise of a single amplifier, while recirculating the signal in one amplifier in order to square the gain (double in dB) yields a $1/f$ noise 6 dB higher, with a net difference of +3 dB versus the cascade amplifier. The amplifier $1/f$ noise cannot be observed directly because the amplifier is in the oscillator loop. The flicker noise b_{-1}/f of the sustaining amplifier is transformed into frequency flicker through the Leeson effect and shows up as a term b_{-3}/f^3 in the phase noise spectrum. In the case of the delay-line oscillator [2, Chapter 5], the Leeson effect takes the form $(b_{-3})_{\text{osc}} = \frac{1}{4\pi^2\tau^2}(b_{-1})_{\text{ampli}}$, where τ is the oscillator roundtrip delay (20 μs in our case). Plugging the amplifier $1/f$ noise in the Leeson effect, we expect that the $1/f^3$ noise is 3 dB higher if the two cascaded amplifiers are replaced with one amplifier using regeneration. This is exactly what was observed, shown in Fig. 8.

5 After fact

This work spins off a long-term research program on high-end oscillators and on frequency synthesis mainly for metrology and for military and space applications. The measurements reported here were done between 2005 and 2009 in different contexts. In the oscillator business, people are obsessed by PM

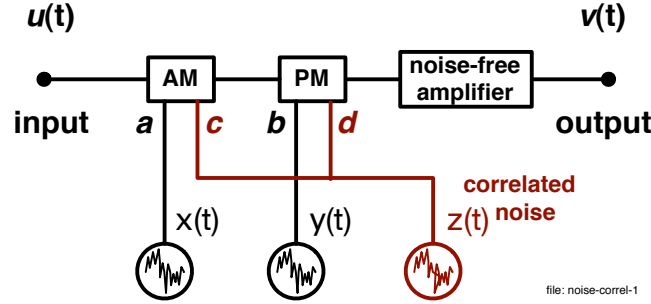


Figure 9: Generalized AM-PM noise model of the amplifier.

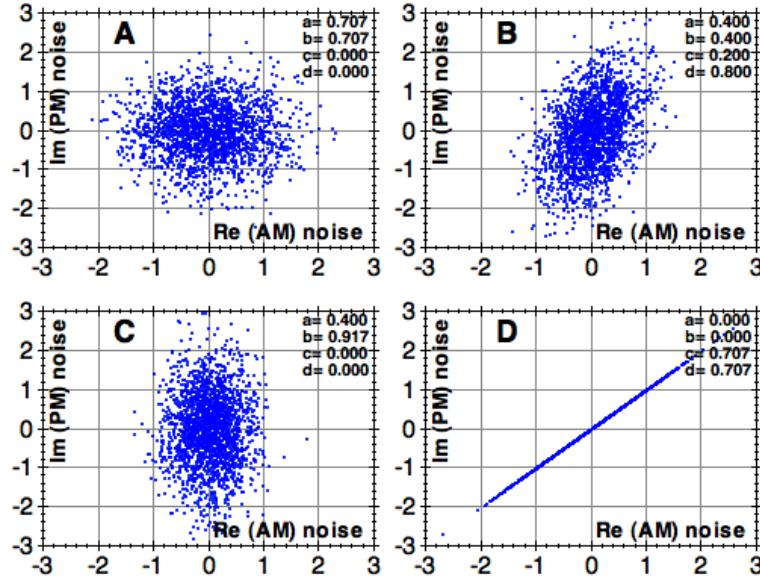


Figure 10: Simulates parametric noise, as it would be detected with the I-Q version of the NMS shown in Fig. 4(B). Dropping an arbitrary scale factor and linearizing for low noise, the real part is the AM noise and the imaginary part is the PM noise. The coefficient a , b , c , d are those of Fig. 9.

noise, while AM noise is considered a scientific curiosity and mentioned only for completeness. Amplitude noise is sometimes measured carefully [51], yet as it belonged to a separate world, or catches the attention because of its detrimental effect on phase-noise measurements [52]. It was only at the time of writing that evidence popped into our mind that parametric AM and PM noises are partially correlated, and therefore that the amplifier noise is best described as in Fig. 9. The necessity for this model is justified by the physics of the most popular amplifier devices. In a bipolar transistor, the fluctuation of the carriers in the base region acts on the base thickness, thus on the gain, and on the capacitance of the reverse-biased base-collector junction. Of course, a capacitance fluctuation impacts on phase. In a field-effect transistor, the fluctuation of the

carriers in the channel acts on the drain-source current, and also on the gate-channel capacitance because the distance between the ‘electrodes’ is affected by the channel thickness. In a laser amplifier, the fluctuation of the pump power acts on the density of the excited atoms, and in turn on gain, on maximum power, and on refraction index. In all these examples AM and PM fluctuations are correlated because both originate from a single near-dc random process.

Since the noise measurements are now terminated or put on hold, and instruments and components scattered in the lab, we can only support the model of Fig. 9 with simulations. In all simulations we normalize on the carrier power, we linearize for low noise, and we set $a^2 + b^2 + c^2 + d^2 = 1$ so that the noise power is in all cases equal to one. The result (Fig. 10) is a scatter plot in which each point is a realization of noise, all at the same Fourier frequency of the AM-PM noise spectrum. The noise is ‘measured’ (actually simulated) by the two-channel version of the NMS shown in Fig. 4(B), where we detect simultaneously the real and the imaginary part with a I-Q mixer [44].

In the case of additive noise, the noise is a Gaussian process of power equally split into the real and imaginary part. This is the symmetric two-dimensional Gaussian distribution shown in Fig. 10(A). We apologize for the aspect ratio not equal to one.

If additive noise is not equally split between AM and PM, as for example it happens when the amplifier is the power compression region, there results an asymmetric Gaussian distribution (Fig. 10(C)). The perfectly saturated amplifier has no AM noise. The scatter plot, not shown, is a vertical line.

Fig. 10(B) shows the case of flicker noise of an amplifier operated in the compression region. The amount of AM and PM is not the same, and there is some correlation between AM and PM noise. For comparison, the plot of Fig. 10(D) represents a (unrealistic) amplifier in which AM and PM noise originates from a single random process with the same modulation efficiency.

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